Debt Sustainability Analysis at the UPB

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UPB definition of public debt sustainability

- Definition of sustainability of public finances used by the UPB ➔ possibility for the government to maintain the current fiscal policy stance without causing a continuous and potentially explosive increase of debt as a percent of GDP in the medium term

- Three types of monitoring instruments:1) analysis of the trajectory of the debt/GDP ratio over the medium term in a baseline scenario; 2) simulations that allow to analyze the sensitivity of this scenario to alternative hypotheses; 3) stochastic analysis (fan chart)

- Baseline ➔ fiscal strategy in government's documents (up to 4 years) + constant structural primary surplus in the medium term (up to 10 years)

Debt dynamics equations

$$D_t = D_{t-1} - OB_t + SFA_t$$

$$D_t = D_{t-1} - PB_t + IE_t + SFA_t$$

$$D_t = (1 + i_t)D_{t-1} - PB_t + SFA_t$$

$$d_t = \frac{(1 + i_t)}{(1 + g_t)} d_{t-1} - pb_t + sfa_t$$

1. Deterministic projections and sensitivity analysis

- Medium-term projections ➔ trajectory of the debt/GDP ratio over a ten-year period based on assumptions about the variables that influence debt dynamics:

$$d_t = \frac{(1+i_t)}{(1+r_t)(1+\pi_t)} d_{t-1} - pb_t + sfa_t$$

- Different issues of fixed- and variable-rate bonds, with short- and long-term maturity, make it difficult to identify the «representative» interest rate
- Simplest assumption ➔ average cost of debt $i_t = IE_t/D_{t-1}$
- UPB assumption ➔distinction between average cost of short-term debt (<= 1 year + indexed) and long-term debt (> 1 year); for the latter further distinction between the average cost of long-term debt falling due and therefore renewed during the year and the average cost of long-term debt not falling due

Interest expenditure

- To illustrate the interest expenditure projections, it is therefore necessary to define some additional variables:

$\gamma_t^s + \gamma_t^l = 1$ - shares of short-term debt (s) and long-term debt (l)

$\gamma_t^{lm} + \gamma_t^{lnm} = \gamma_t^l$ - shares of long-term debt falling due and renewed within the year (lm) and not falling due within the year (lnm)

- Decompose interest expenditure in that due to: 1) short-term debt; 2) long-term debt falling due and renewed within the year; and 3) long-term not falling due:

$$IE_t = IE_t^s + IE_t^{lm} + IE_t^{lnm}$$

Interest expenditure

- It follows that:

$$IE_t = i_t^s \cdot \gamma_t^s \cdot (D_{t-1} + \Delta D_t) + i_t^l \left(\gamma_t^{lm} \cdot D_{t-1} + \gamma_t^l \cdot \Delta D_t\right) + i_t^{lnm} \cdot \gamma_t^{lnm} \cdot D_{t-1}$$

where the average cost of short-term debt (i_t^s) and the average cost of long-term debt falling due and renewed within the year (i_t^l) are estimated through market interest rates

- For the implicit average cost of long-term debt not falling due(i_t^{lnm}), the European Commission suggests an estimate through the equation:

$$i_t^{lnm} \approx w_t \cdot i_{t-1}^{lnm} + (1 - w_t) \cdot i_{t-1}^l$$

where:

$$w_t = \frac{\gamma_t^{lnm} D_{t-2}}{\gamma_t^l D_{t-1}}$$

- This formula makes it possible to obtain a lasting effect on interest expenditure from changes of the long-term interest rate

Recursive solution of the deterministic scenario

$$w_t = \frac{\gamma_t^{lnm} D_{t-2}}{\gamma_t^{l} D_{t-1}}$$

$$i_t^{lnm} \approx w_t \cdot i_{t-1}^{lnm} + (1 - w_t) \cdot i_{t-1}^{l}$$

$$ie_{\mathrm{t}} = \frac{1}{1 - \mathrm{i}_{\mathrm{t}}^{s} \cdot \gamma_t^{s} - \mathrm{i}_{\mathrm{t}}^{l} \cdot \gamma_t^{l}} \cdot \Bigg[\left(\mathrm{i}_{\mathrm{t}}^{s} \cdot \gamma_t^{s} + \mathrm{i}_{\mathrm{t}}^{l} \cdot \gamma_t^{lm} + \mathrm{i}_{\mathrm{t}}^{lnm} \cdot \gamma_t^{lnm} \right) \cdot \frac{\mathrm{d}_{t-1}}{(1 + r_{\mathrm{t}})(1 + \pi_{\mathrm{t}})} - \left(\mathrm{i}_{\mathrm{t}}^{s} \cdot \gamma_t^{s} + \mathrm{i}_{\mathrm{t}}^{l} \cdot \gamma_t^{l} \right) \cdot (\mathrm{pb}_{\mathrm{t}} - \mathrm{sfa}_{\mathrm{t}}) \Bigg]$$

$$d_t = \frac{d_{t-1}}{(1 + r_t)(1 + \pi_t)} + ie_t - pb_t + sfa_t$$

Assumptions on the deterministic scenario

Variable	Up to 2022	Beyond 2022
r_t	DEF forecast	Gradual convergence to 0,5%
π_t	DEF forecast	Gradual convergence to 2%
i_t^s	DEF forecast/Spot and forward interest rates	Gradual convergence to 3%
i_t^l	DEF forecast/Spot and forward interest rates	Gradual convergence to 4.5%
pb_t	DEF forecast	Constant structural primary balance
sfa_t	DEF forecast	Zero

A medium-term projection of the baseline scenario is thus obtained, which in our exercises -- at least until now -- shows a decreasing profile in the medium-term, as the primary balance offsets the unfavorable impact of the i-πr differential («snowball»).

Sensitivity analysis

- Sensitivity analyses are then performed to assess the risk that the DEF scenario, generally relatively favourable, may actually no longer be so after (realistic) changes to the underlying assumptions

- Exercises usually include:

1) sensitivity wrt macroeconomic forecast (nominal GDP growth from UPB)

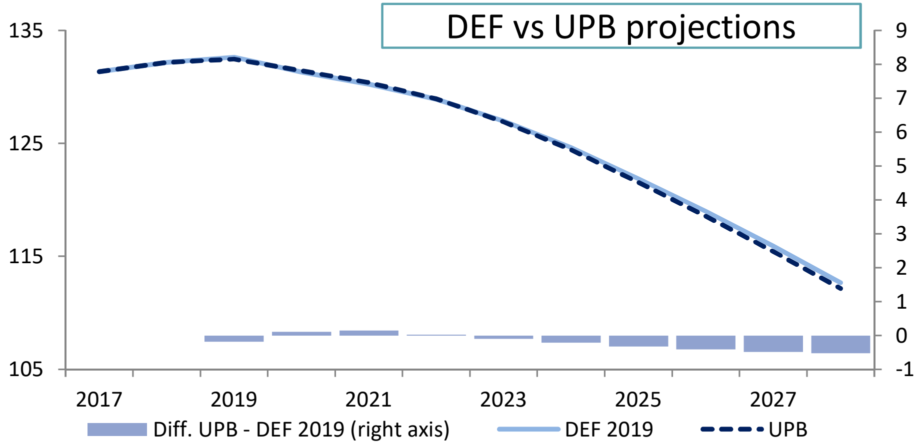
2) sensitivity wrt the primary balance (constant structural primary balance from 2019)

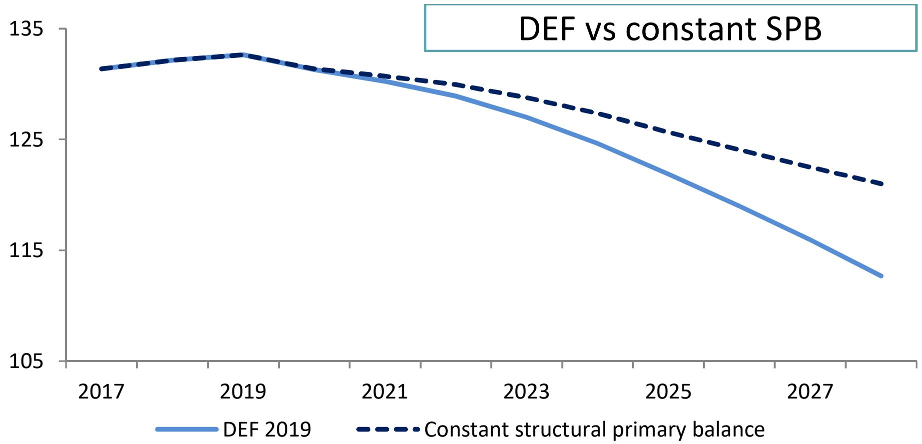
3) sensitivity wrt different fiscal assumptions (e.g. no activation of VAT safeguard clauses)

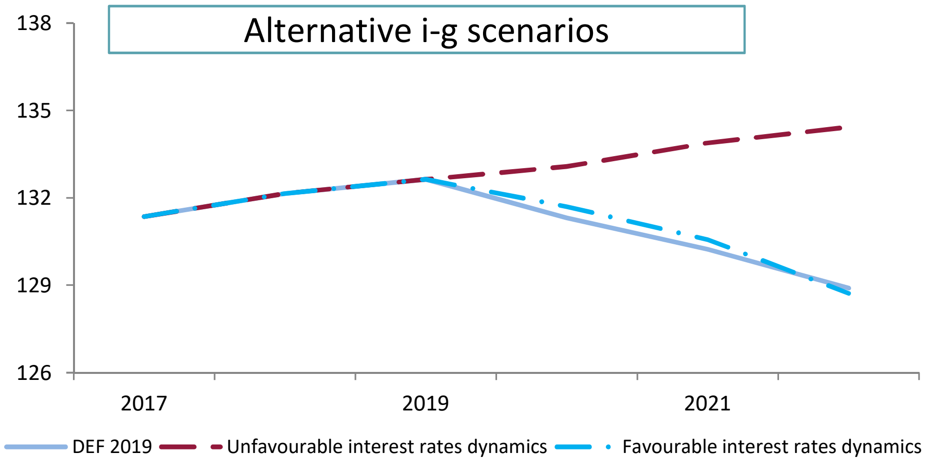
4) Sensitivity wrt i-πr differential

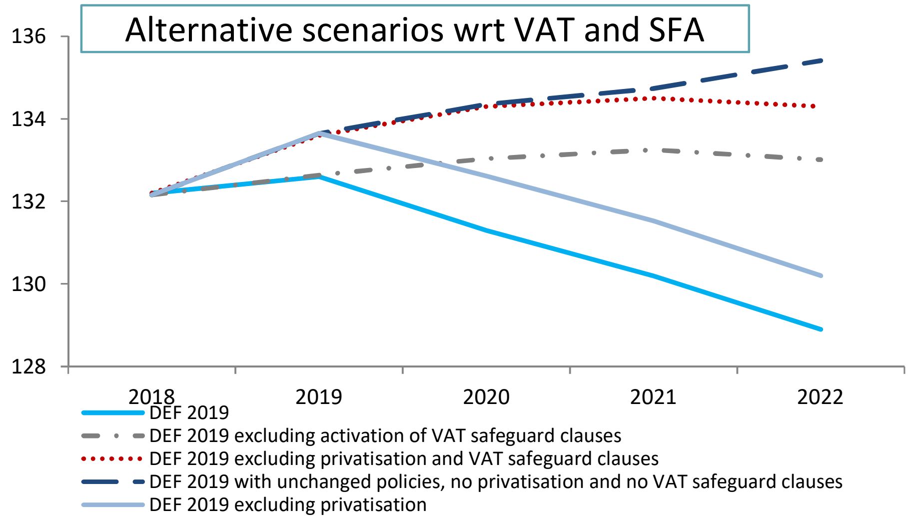
- The exercises take into account the average elasticity of the primary balance wrt the cycle (0.544 = EU estimate), the average budget multipliers (UPB model), the pass-trough of π over i (half, ad hoc hypothesis)

Sensitivity analysis









2. Stochastic simulations

- The stochastic analysis of the trajectory of the debt / GDP ratio is a natural generalization of the sensitivity analysis

- Through Monte Carlo techniques, a large number of scenarios is generated, which allows the construction of a fan chart for the trajectory of the debt/GDP ratio

- Stochastic scenarios are obtained by generating innovations to the determinants of the debt dynamics equation

- Innovations are generated by considering the variance-covariance matrix of (the difference of) the same determinants estimated with past data

Definition and estimation of the shocks

- Take the determinants of the debt dynamics equation:
  - ✓ r_t, real GDP growth rate
  - ✓ π_t, GDP deflator growth rate
  - ✓ i_t^s, short-term interest rate
  - ✓ $spread_t = i_t^l - i_t^s$, long/short-term interest rate differential
- Innovations (on a quarterly basis) are defined on the basis of historical series from 1990T1 to 2018T4 as

$$s_t^x = x_t - x_{t-1}, \text{ dove } x_t \in (r_t, \pi_t, i_t^s, spread_t)$$

- Define S the vector containing all the shocks to the variables x, with zero mean and variance-covariance matrix $Var\ (S) = \Sigma$
- Through Monte-Carlo simulations, obtain a vector E of shocks with a multivariate normal distribution $E \sim N\ (0, \Sigma)$; where ε_t^x is the shock relative to the variable x for the period t
- The procedure is repeated 5,000 times to obtain as many vectors of innovations; quarterly shocks are reported on a yearly basis as simple sums

Estimation of the shocked variables

- The "shocked" variables are then defined equal to their baseline value plus the shock:

$$\overline{x_t} = x_t + \varepsilon_t^x$$

- The shocks so defined are temporary in nature as they have no lasting effect over time

- The variables are subject to the shock for only one period, returning to their baseline value in the following period to be subject to a new independent shock

- The variables therefore take on value:

$$\overline{x_t} = x_t + \varepsilon_t^x \text{ al tempo t, e}$$
$$\overline{x_{t+1}} = x_{t+1} + \varepsilon_{t+1}^x \text{ al tempo t+1}$$

Permanent shocks to interest rates

- In an alternative scenario, interest rates are also subject to shocks with long-lasting or permanent effects

- In this scenario, short-term interest rates diverge substantially from the baseline trajectory and are guided by the sequence of shocks (unit root):

$$\begin{cases} \overline{\overline{i^{s}_{T0}}} = i^{s}_{T0} + \varepsilon^{i^{s}}_{T0}\,, nel\ primo\ periodo, T0 \\ \overline{\overline{i^{s}_{t}}} = \overline{\overline{i^{s}_{t-1}}} + \varepsilon^{i^{s}}_{t}, per\ t > T0 \end{cases}$$

- Il tasso di interesse a medio-lungo termine soggetto a shock permanenti viene definito come:

$$\overline{\overline{i^{lm}_{t}}} = \overline{\overline{i^{s}_{t}}} + \overline{spread_{t}}$$

Simulation of the primary balance

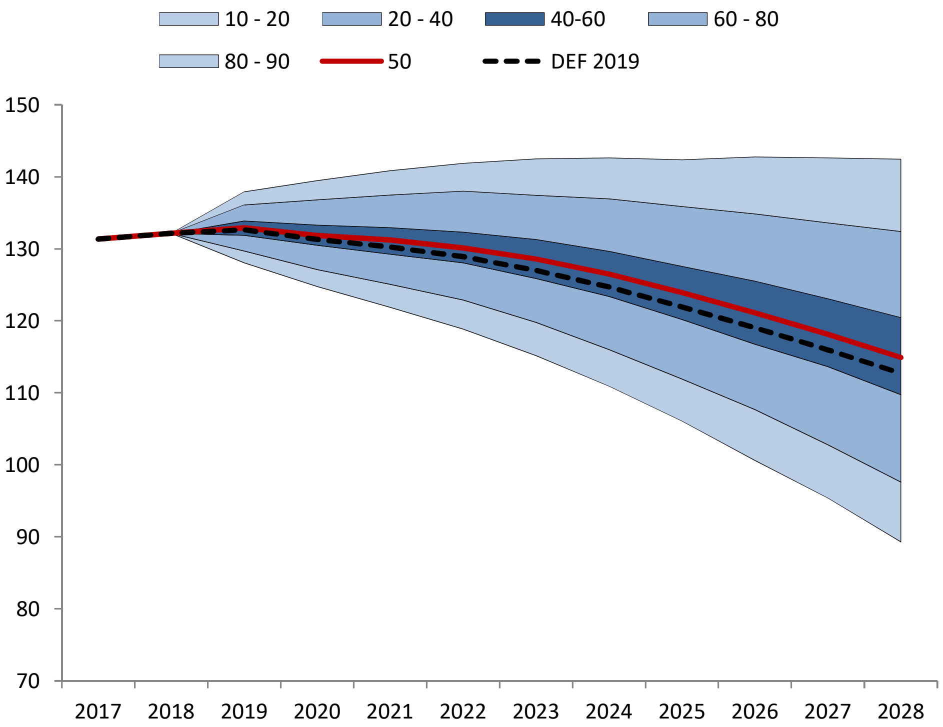
- The primary balance is not subject to direct shocks, but is indirectly influenced by shocks to the real GDP growth rate through the elasticity of the balance with respect to the growth rate itself (m = 0.544)

- Note that shocks to the real growth rate have a permanent impact on the primary balance as a percentage of GDP:

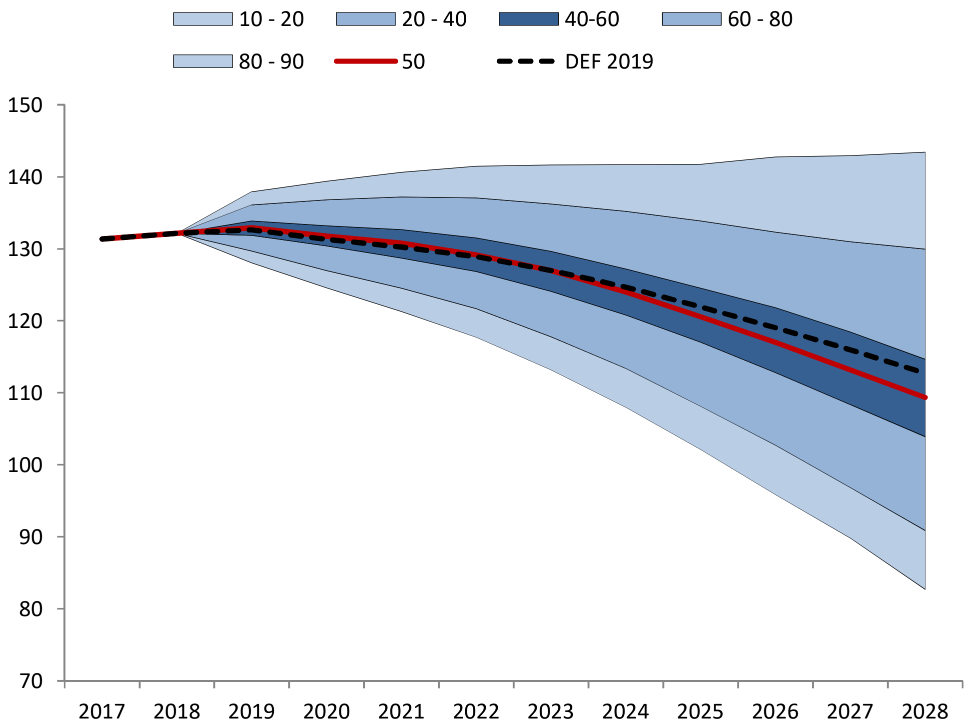
$$\begin{cases} \overline{\overline{pb_{T0}}} = pb_{T0} + m \cdot \varepsilon^{r}_{T0}\,, \text{nel primo periodo, T0} \\ \overline{\overline{pb_t}} = pb_t + m \cdot \sum_{T0}^{t} \varepsilon^{r}_{t}\,, per\ t > T0 \end{cases}$$

Stochastic simulations – results

Fan chart debt/GDP, temporary shocks

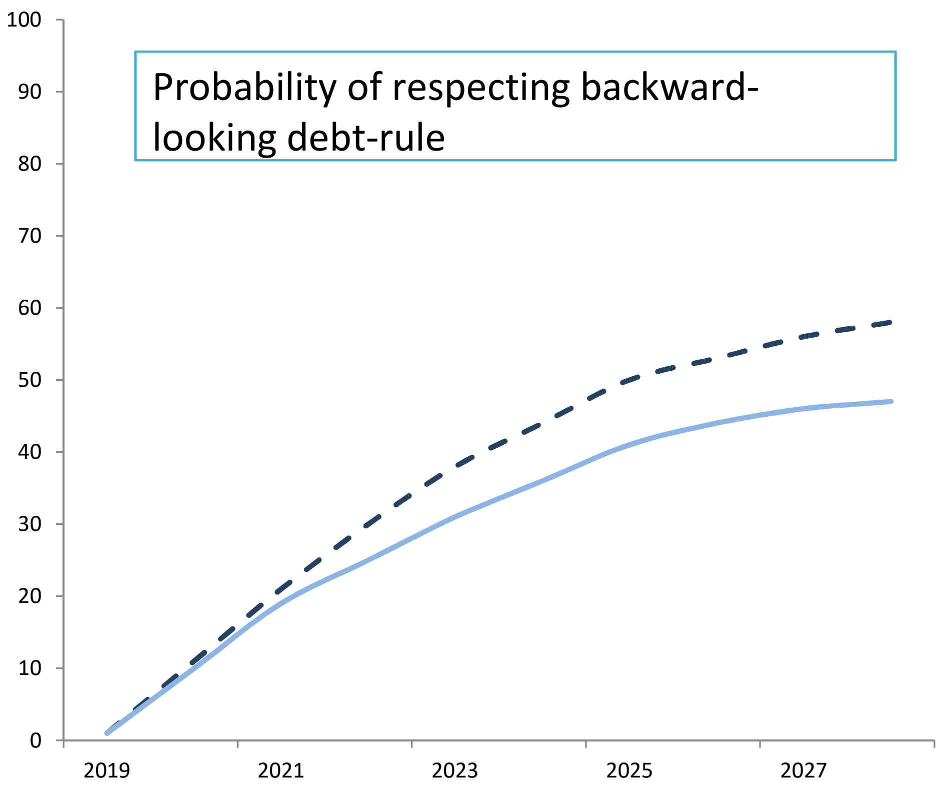


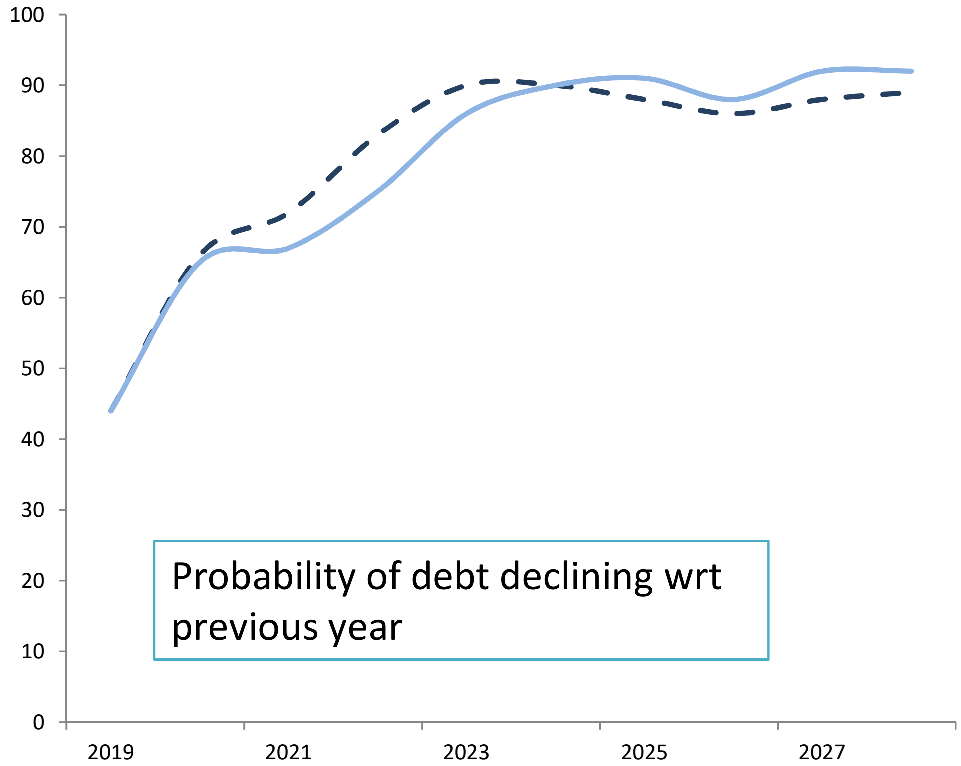
Fan chart debt/GDP, permanent shocks



The black lines show (from bottom to top) the 10^{th}, 20^{th}, 40^{th}, 60^{th}, 80^{th}, 90^{th} percentiles. The red line the median, and the black dotted line shows the DEF trajectory

Stochastic simulations - results





Dashed lines: permanent shocks Solid lines: temporary shocks

Future work avenues

- Considering in the simulation also the elasticity of the primary balance wrt inflation

- Further disaggregation of interest expenditure:

1. decomposing interest expenditure on short-term debt and inflation-linked debt
2. decomposing interest expenditure on 2, 3, 5, 7, 10, 30 year debt
3. separate modeling of interest expenditure on debt other than State securities

- Endogenization of some or all of the variables that determine the dynamics of debt ("fiscal VAR model" with debt dynamics)

Main references

European Commission, «Assessing Public Debt Sustainability in EU Member States: A Guide», European Economy Occasional Papers 200, September 2014

Julio Escolano, «A Practical Guide to Public Debt Dynamics, Fiscal Sustainability, and Cyclical Adjustment of Budgetary Aggregates", IMF Technical Notes and Manuals, January 2010

Rapporto dell'Ufficio parlamentare di bilancio sulla programmazione di bilancio 2016:

http://www.upbilancio.it/rapporto-sulla-programmazione-di-bilancio-2016-2/

Appendix - Notations

t	- Year
t_0	- Latest observed year
d_t	- Debt ratio
pb_t	- Primary balance ratio
ie_t	- Interest rate ratio
sfa_t	- Stock-flow adjustment ratio
π_t	- GDP deflator growth rate
i_t	- Average cost of debt (nominal) = IE_t/D_{t-1}
r_t	- Real GDP growth rate
g_t	- Nominal GDP growth rate